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THE TRANSPORTATION PROBLEM UNDER PROBABILISTIC AND FUZZY UNCERTAINTIES

The paper presents further development of an approach proposed by Stefan Chanas and Dorota Kuchta [1], [2] for the transportation problem solution in the case of fuzzy coefficients. The direct fuzzy extension of usual simplex method is used to realize a numerical fuzzy optimization algorithm with fuzzy constraints. It must be emphasized that the fuzzy numerical method proposed is based on the practical embodiment of the pioneer idea of Stefan Chanas [3], [4] to consider fuzzy values in the probabilistic sense. The problem is formulated in a more general form of the distributor's benefit maximization.

1. Introduction

The task of optimizing distributor's decisions can be reformulated as the generalization of a classical transportation problem. The conventional transportation problem is the special type of a linear programming problem where special mathematical structure of restrictions is used. In classical approach, transportation costs from M wholesalers to N consumers are minimized.

In 1979, Isermann [7] introduced an algorithm for solving this problem, which provides effective solutions. Ringuest and Rinks [10] proposed two iterative algorithms for solving linear, multicriterial transportation problem. A similar solution is proposed in [5].

The different effective algorithms have been worked out for this transportation problem, with appropriate parameters of a task being described in the form of real numbers. Nevertheless, such conditions are seldom or almost never fulfilled because of natural uncertainties we meet in real-world problems. For example, it is hard to

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define stable cost of specified route. In work [6], this problem was solved for the case of interval uncertainty of transportation costs.

In works by S. Chanas and D. Kuchta [1], [2], the approach based on interval and fuzzy coefficients had been elaborated. The further development of this approach has been introduced in work [11].

All the above mentioned works introduce restrictions in the form of a membership function. This allows the initial fuzzy linear programming problem to be transformed into the net of usual linear programming tasks by use of the well defined analytic procedures. However, in practice the membership functions, describing uncertain parameters of the models used can have highly complicated forms. In such cases, the numerical approach is needed.

The main technical problem when constructing a numerical fuzzy optimization algorithm is the comparison of fuzzy values. To deal with this problem, we use the approach proposed in [8], [9] and well described in [12], which is based on α-level representation of fuzzy numbers and probability estimation of the fact that a given interval is greater than/equal to another one. We note that probabilistic approach was used only to infer the set of formulae for deterministic quantitative estimation of the inequality/equality of intervals. The method allows to compare the interval and real number and to take into account (implicitly) the widths of intervals ordered.

The approach put forward allows us to accomplish the direct fuzzy extension of classical numerical simplex method with its implementation using the tools of object-oriented programming.

2. Description of the method

In the approach proposed we not only minimize the transportation costs but in addition we maximize the distributor's profits under the same conditions.

The distributor deals with *M* wholesalers and *N* consumers (see Fig. 1).

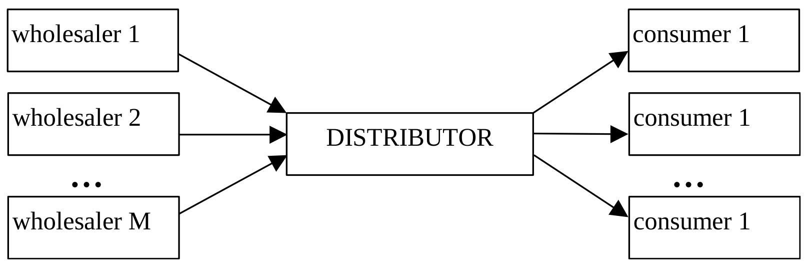

Fig. 1. The scheme of distributor’s activity

Let $a_{i,}$, i = 1 ... M, be the maximal quantities of goods that can be offered by wholesalers and b_j, j = 1 ... N, be the maximal requirements of consumers as regards goods. In accordance with the contracts signed a distributor must buy at least p_i units of goods at a price of t_i monetary units for a unit of goods from each i-th wholesaler and to sell at least q_j goods units at a price of s_j monetary units for a unit of goods to each j-th consumer. The total transportation cost of delivering good unit from the i-th wholesaler to the j-th consumer is denoted by c_{ij}.

There are reduced prices k_i for a distributor who buys greater quantities of goods than stipulated in contract quantities p_i and also the reduced prices r_j for consumers who buy greater quantities of goods than contracted q_j. The problem is to find optimal quantities of goods x_{ij} (i = 1, ..., M; j = 1, ..., N) delivered from the i-th wholesaler to the j-th consumer maximizing the distributor's total benefit D under restrictions. Assuming that all the above mentioned parameters are fuzzy ones the, resulting optimization task is formulated as:

$$\hat{D} = \sum_{i=1}^{M} \sum_{j=1}^{N} \left(\hat{z}_{ij} * \hat{x}_{ij} \right) \rightarrow \max , \tag{1}$$

$$\sum_{j=1}^{N} \hat{x}_{ij} \leq \hat{a}_i \ (i = 1 \ldots M) , \qquad \sum_{i=1}^{M} \hat{x}_{ij} \leq \hat{b}_j \ (j = 1 \ldots N) , \tag{2}$$

$$\sum_{j=1}^{N} \hat{x}_{ij} \geq \hat{p}_i \ (i = 1 \ldots M) , \qquad \sum_{i=1}^{M} \hat{x}_{ij} \geq \hat{q}_j \ (j = 1 \ldots N) , \tag{3}$$

where $\hat{z}_{ij} = \hat{r}_j - \hat{k}_i - \hat{c}_{ij}$ (i = 1, ..., M; j = 1, ..., N) and $\hat{D}, \hat{z}_{ij}, \hat{a}, \hat{b}, \hat{q}, \hat{p}$ are fuzzy values.

To solve the problem (1)–(3), a numerical method based on the α-cut representation of fuzzy numbers and probabilistic approach to the interval and fuzzy interval comparison has been elaborated. The direct fuzzy extension of usual simplex method is applied. The use of object-programming tools makes it possible to get the results of fuzzy optimization, i.e., $\hat{x}_{ij}$, in the form of fuzzy numbers as well.

To estimate the effectiveness of the method proposed, the results of fuzzy optimization were compared with those obtained from (1)–(3) when all the uncertain parameters were considered as normally distributed random values. Of course, in the last case all the parameters in (1)–(3) were considered as real numbers.

To make the results we got using the fuzzy and probability approaches comparable, a special simple method for transformation frequency distributions into fuzzy numbers without losts of useful information was employed to achieve the comparability of uncertain initial data in fuzzy and random cases.

In practice, we often have a problem with varying precision of representing the uncertain data we use. For instance, one part of parameters used can be represented in form of trapezoid fuzzy numbers based on of the expert's opinions and at the same time, the other part of them can have the form of a histogram or frequency distributions of considerable complicated form we got as a result of statistical analyses.

In these cases, the methodologically correct approach is to transform all the uncertain data available to the form of the smallest certain level we met in our task. Thus, we have to transform the data represented in form of frequency distributions or histogram to the membership functions of fuzzy numbers.

To present the initial data in fuzzy number form, first we should apply an algorithm, which builds the membership function on the basis of frequency distribution, if such exists, or directly using a histogram.

In the simplest case of normal frequency distributions, they can be exhaustively described by their averages m and standard deviations σ. In the more complicated situations it seems better to use directly the histograms.

That is why, we use the numerical algorithm which allows us to transform the frequency distribution or histogram to trapezoidal fuzzy number.

As an illustration, let us consider the reduction of frequency distribution (see Fig. 2) to the fuzzy number.

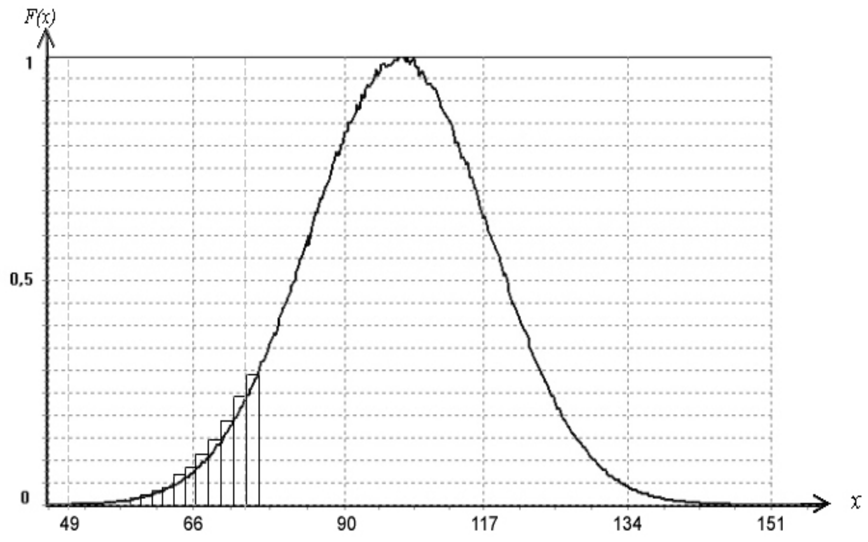

Fig. 2. Frequency distributions to be transformed

We have to accomplish the following steps of the algorithm:

Step 1. In the interval within the smallest value $x_{\min}$ (in our example, $x_{\min} = 50$) and maximum value $x_{\max} = 151$ we define the function $F(x_i)$ as surface area under the curve (see Fig. 2) from $x_{\min}$ to current x_i. As a result, we get a cumulative function shown in Fig. 3. It is easy to see that function $F(x)$ is, in fact, the probability of $x < x_i$.

Step 2. Using the cumulative function $F(x)$ we ask the decision-makers (experts) for the four values $F(x_i)$ $i = 0, ..., 3$, which define the mapping of $F(x)$ on X in such a way that they provide the lower and upper α-levels of trapezoidal fuzzy number (the top and bottom trust levels of fuzzy number). In our example (see Fig. 3), the intervals [95], [105] and [78], [120] are in essence the 30% and 90% probability confidence intervals.

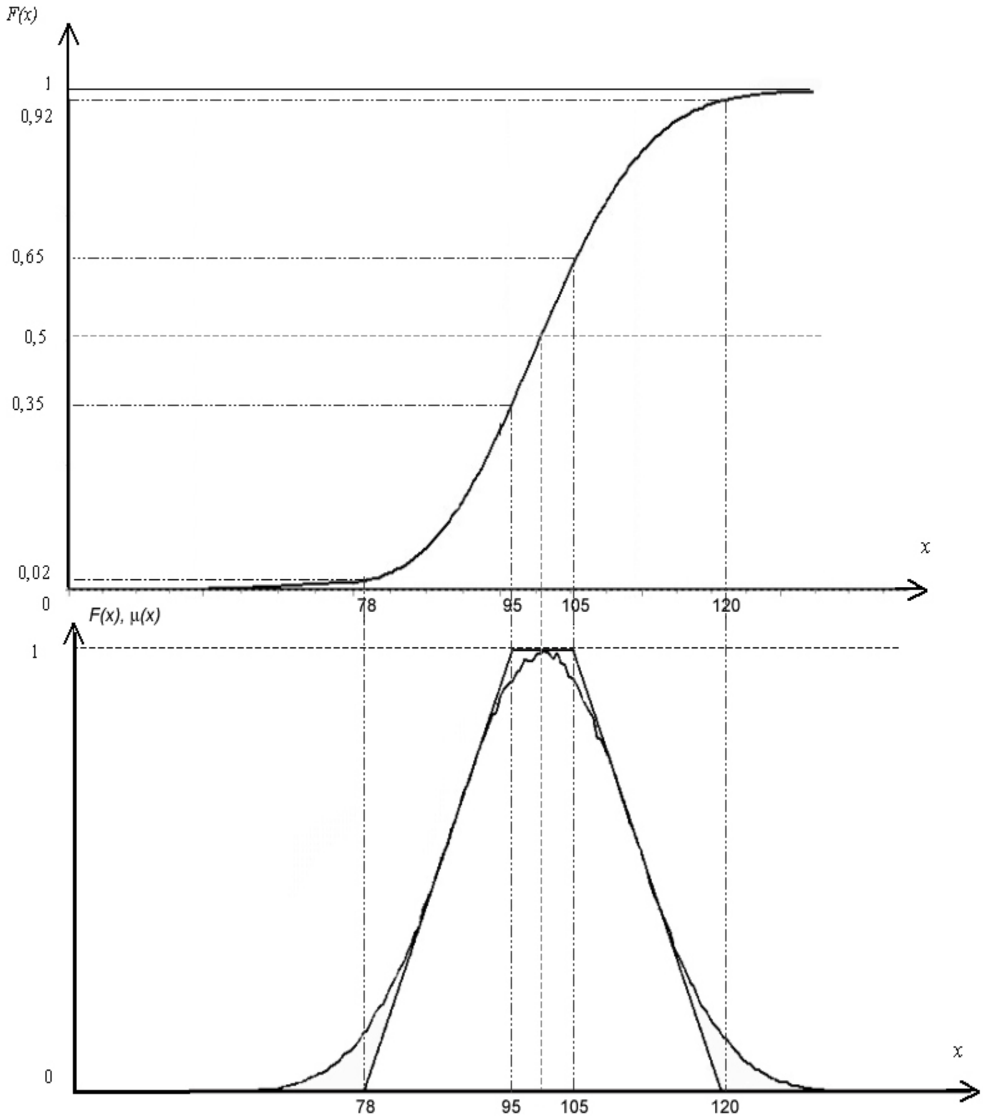

Fig. 3. The transformation of cumulative function to a fuzzy number

As a result, we get the trapezoidal fuzzy interval represented in our example by quadruple [78], [95], [105], [120].

Is easy to see that transformation accuracy depends only on the expert's opinion about suitability and correctness of upper and lower confidence intervals chosen.

It is worth noting that the main advantage of the method presented is that it can be successfully used in both cases: when we have the initial data in the form of frequency distribution function and in the form of a rough histogram.

The method allows us to represent all the uncertain data in a uniform way as trapezoid fuzzy intervals.

In the method of fuzzy programming problem (1)–(3) solution all fuzzy numbers are represented as the sets of α-cuts. In fact, this reduces fuzzy problem into the set of crisp interval optimization tasks.

The final solution has been obtained numerically with the use of probabilistic approach to interval comparison. The interval arithmetic rules needed were established with the help of object-oriented programming tools.

The standard Monte-Carlo procedure was used in the probability approach to the description of uncertain parameters of the optimization task (1)–(3). In fact, for each randomly selected set of real valued parameters of task (1)–(3) we solve the usual linear programming problem.

3. Numerical example

To compare the results of fuzzy programming with those obtained when using the Monte-Carlo method, all the uncertain parameters were previously expressed as by Gaussian frequency distributions. Their averages are presented in Table 1. For simplicity, all the standard deviations σ were accepted as equal to 10 i.m.

Table 1

Average values of Gaussian distributions of uncertain parameters

$a_1 = 460$	$b_1 = 410$	$p_1 = 440$	$q_1 = 390$	$t_1 = 600$	$s_1 = 1000$	$k_1 = 590$	$r_1 = 990$
$a_2 = 460$	$b_2 = 510$	$p_2 = 440$	$q_2 = 490$	$t_2 = 491$	$s_2 = 1130$	$k_2 = 480$	$r_2 = 1100$
$a_3 = 610$	$b_3 = 610$	$p_3 = 590$	$q_3 = 590$	$t_3 = 581$	$s_3 = 1197$	$k_3 = 570$	$r_3 = 1180$
$c_{11} = 100$	$c_{12} = 30$	$c_{13} = 100$					
$c_{21} = 110$	$c_{22} = 36$	$c_{23} = 405$					
$c_{31} = 120$	$c_{32} = 148$	$c_{33} = 11$					

The results we obtained with the use of fuzzy optimization method and Monte--Carlo method (usual linear programming with real valued but random parameters) are presented in Figs. 4–8 for the case $M = N = 3$, where the final frequency distributions F are marked by dotted lines, and fuzzy numbers μ a solid lines.

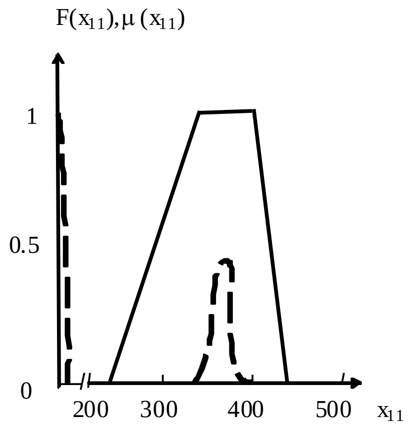

Fig. 4. Frequency distribution F and fuzzy number μ for optimized x_{11}

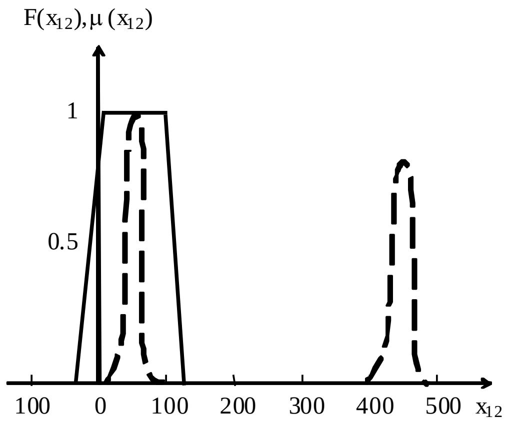

Fig. 5. Frequency distribution F and fuzzy number μ for optimized x_{12}

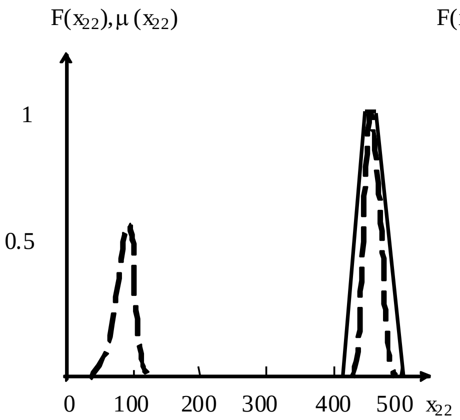

Fig. 6. Frequency distribution F and fuzzy number μ for optimized x_{22}

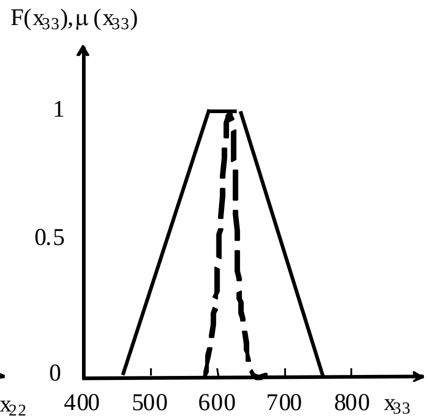

Fig. 7. Frequency distribution F and fuzzy number μ for optimized x_{33}

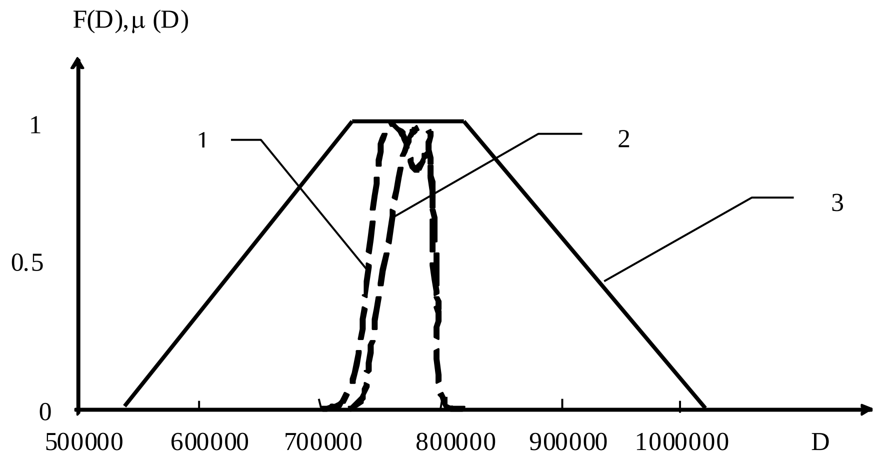

Fig. 8. Frequency distribution *F* and fuzzy number μ for optimized benefit *D*:
1 – Monte-Carlo method for 10 000 random steps;
2 – Monte-Carlo method for 100 000 000 random steps;
3 – Fuzzy approach

It is easy to see that the fuzzy approach gives us some more wider fuzzy intervals than Monte-Carlo method. It is interesting to see that using probabilistic method we can get even two-extreme results whereas fuzzy approach always gives us results without ambiguity. It is worth noting that probabilistic method demands too many random steps (about 100 000 000) to obtain the smooth frequency distribution of resulting benefit *D*. Thus, it seems rather senseless to use this method in practice.

Summary

A direct numerical method for solving fuzzy transportation problem is elaborated. The method is based on α-level representation of fuzzy numbers and probability estimation of the fact that a given interval is greater than/equal to another interval (this idea was firstly put forward by S. Chanas). The approach proposed makes it possible to accomplish the direct fuzzy extension of usual simplex method.

The results of case studies with the use of fuzzy optimization method and Monte-Carlo method (usual linear programming with real valued but random parameters) show that the fuzzy approach has considerable advantages in comparison with Monte-Carlo method, especially from the computational point of view.

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Problem transportowy w warunkach niepewności probabilistycznej oraz rozmytej

Zaprezentowano dalsze rozwinięcie podejścia proponowanego przez Profesora Stefana Chanasa i Dorotę Kuchtę w rozwiązaniu problemu transportowego w przypadku zastosowania rozmytych współczynników. Zagadnienie optymalizacji decyzji dystrybutora zostało sformułowane jako uogólnienie klasycznego problemu transportowego, w tym przypadku jako zagadnienie rozmytego, nieliniowego, dwukryterialnego programowania. Bezpośrednie rozmyte rozszerzenie zwykłej metody Simplex zostało użyte w celu wypracowania rozmytego algorytmu numerycznego, zawierającego rozmyte ograniczenia. Należy podkreślić, że zaproponowana rozmyta metoda numeryczna jest oparta na praktycznym ucieleśnieniu pionierskiej idei Stefana Chanasa, dotyczącej porównywania liczb rozmytych w sensie probabilistycznym. W celu porównania rezultatów uzyskanych przy zastosowaniu programowania rozmytego z uzyskanymi metodą Monte-Carlo, przedstawiony został przykład numeryczny zawierający interesujące wyniki.